

Study on Mathematics Model of Leather Creeping Behavior in Lastometer Deformation States

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Abstract: Leather is one kind of cross-linking collagen protein. According to experimental result that creeping behaviors of leather do not keep to the Kelvin model, it is obviously that leather is one kind of non-linear viscoelastic material. The generalized Kelvin model is obtained after the Kelvin model to be amended; it kept to the creeping experiment data of leather through mathematic simulation (all significant levels of model are more than 99%). The generalized Kelvin model can be used to describe the leather creeping behavior in deformation states of the lastometer successfully. The research works also respectively taken 2~4 Kelvin units in series as a mechanical model, respectively taken 2~4 generalized Kelvin units in series as a mechanical model, and respectively taken other 6 functions as a mechanical model, and carried through mathematic simulation to every mechanical model with the creeping experiment data of leather, and then examined the relative coefficient(R) of every model.

Key words: leather; creep behavior; mechanical model; lastometer

1 Introduction

As a biopolymer composite, leather behaviors a visco-elastic character, which means that the shape, area length changes with the increase of the duration of force on leather samples. The viscoelasticity deals closely with shape-stability and comfortable characteristics of leather. Therefore, it is of great significance to study the viscoelasticity of leather.

There are two main measures to study the viscoelasticity of polymeric materials^[1]. The first one, known as stress relaxation behavior, is to study the changes in stress when samples are stretched at a constant strain. The other one is to study the changes in strain when samples are stretched at a constant stress. It is to study the creep behaviors. Creep is a continuous deformation process due to constant loading and an essential property related to viscoelasticity. To studying the creep behavior of materials is an important way to understand the viscoelastic properties of materials.

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With regard to the study on the creep behaviors of leather, only a few studies have been reported before. Keyong Tang and his coworkers^[2] studied the creep of leather by a tensile test, and successfully described the creep behavior of Nappa upper pig leather using three series of Kelvin units. The relations between creep behavior and comfortable characteristics of leather were also discussed in his researches. Manich^[3] designed a lastometer type experiment to study the creep behaviors of leather. Creep index was measured and it was found that the index was related to subjective handle parameter of leather. The present paper aims to study the creep behaviors of leather with a lastometer type measurement^[4] designed and manufactured by ourselves and to describe the creep behaviors by a proper mathematical model.

The mechanical behaviors of elastic solid (springs) obey to Hook's law, and a spring model may be used to describe the mechanical behaviors of elastic solid successfully. The mechanical behaviors of viscous fluid obey to Newton's law, and a dashpot model is usually used to describe the mechanical behaviors of viscous fluid successfully as well^[5]. Neither the spring model nor the dashpot model may describe the mechanical behavior of polymers successfully. The mechanical behaviors of polymers are located between those of the both models. In order to describe the mechanical behaviors of viscoelastic materials, both spring models and dashpot models are needed. An approach to characterising the creep behaviors is to develop mathematical models based on combinations of elastic (spring) elements and viscous (dashpot) elements. In order to describe the mechanical behaviors of viscoelastic materials, both spring models and dashpot models are needed.

The creep behavior of many materials can be simulated by the behavior of a mechanical model with a number of elastic solids and viscous fluids^[6]. Kelvin model, a paralleled of a spring and a dashpot, can successfully describe the creep behavior of polymers with one retardation time. The relation between creep strain $\epsilon(t)$ and creep duration (t) is as

$$\epsilon(t) = \epsilon(\infty)(1 - e^{-t/\tau}) \quad (1)$$

where $\epsilon(\infty)$ is the equilibrium strain of the Kelvin model, and τ is the creep retardation time of the Kelvin model.

Most polymers, however, have a series of retardation times due to the multiplicity of its structural units and the complexity of the molecules movement. Therefore, generalized Kelvin model, a model consisting of some Kelvin units is needed to satisfactorily imitate the creep behavior of materials^[7]. The equation of generalized Kelvin model is:

$$\epsilon(t) = \sum_{i=1}^n \epsilon_i(\infty)(1 - e^{-t/\tau_i}) \quad (2)$$

where $\varepsilon_1(\infty)$, $\varepsilon_2(\infty)$, ..., $\varepsilon_n(\infty)$ is the equilibrium strain of the Kelvin models, respectively, and τ_1 , τ_2 , ..., τ_n is the creep retardation time of the Kelvin models, respectively. Modified generalized Kelvin model^[8] is composed of a spring and a generalized Kelvin model. The equation is:

$$\varepsilon(t) = \varepsilon_0 + \sum_{i=1}^n \varepsilon_i(\infty)(1 - e^{-t/\tau_i}) \quad (3)$$

where ε_0 is the strain of the spring. $\varepsilon_1(\infty)$, $\varepsilon_2(\infty)$, ..., $\varepsilon_n(\infty)$ is the equilibrium strain of the Kelvin models, respectively, and τ_1 , τ_2 , ..., τ_n is the creep retardation time of the Kelvin models, respectively.

2 Experimental

2.1 Materials and Instruments

Cattlehide upper leathers were kindly provided by Xingye leather science and technology PLC, Fujian, China. Lastometer tester was designed and made by ourselves.

2.2 Procedure

Leather samples were air-conditioned at $27 \pm 2^\circ\text{C}$ with the relative humidity of $65 \pm 5\%$ for 48h before experiment was conducted. Standard circular leather samples with the diameter of 50mm were obtained from the official sampling position. A sample was then spread uniformly on the raised table (with a standard 25mm hole in its centre) of our lastometer tester and secured by the clamps. A probe with force was pushed against the grain surface. The time and the variation of distention (h) were recorded during the process. The whole time of creep was 350 seconds. Fig. 1 shows the working mechanism of the gauge developed for the study.

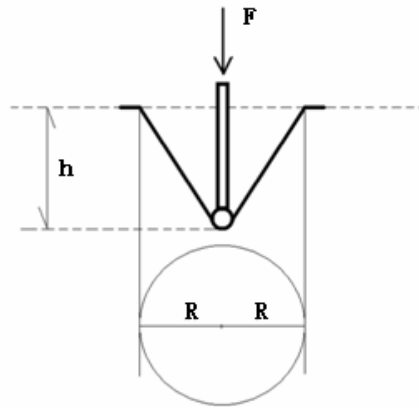


Fig. 1 Working mechanism of the gauge developed for measuring the creep of leather

2.3 Data process

During lastometer testing, the relation among strain (ε), distention (h) and semi-diameter

(R) of the leather sample is according to equation (4):

$$\varepsilon = \frac{\sqrt{h^2 + R^2} - R}{R} \times 100\% \quad (4)$$

The experimental data were fitted by equation (2) and (3) with Originpro7.5 when the study was done.

3 Results and Discussion

3.1 Experimental data of creep

Fig.2 shows the creep curves of leather samples. From Fig.2, we know that the creep develops quickly at the beginning of the experiment and turned slow gradually. The mechanical property is due to the nature of the collagenic fiber network. There are plenty of spare spaces in the network and between collagen molecules^[9]. At the beginning of creep experiment, the cross networks of collagen fibers were extended, and the deformation was increased rapidly. With a quick creep behavior, the needs for comfortable characteristics of leather were met, and it would be thought that the material from which the shoes were made is good. For different people, the foot shapes are different. Creep will start quickly at the moment when shoes are dressed according to the shapes of feet. So leather shoes will provide a comfortable feeling to the people who are dressed. With the increase of time, the strain reaches an equilibrium value gradually. This means that the leather has a good shape-stability. When leather shoes are dressed, the strain will not increase indefinitely, which means the shoes will have a good-looking even they have been dressed for a long time.

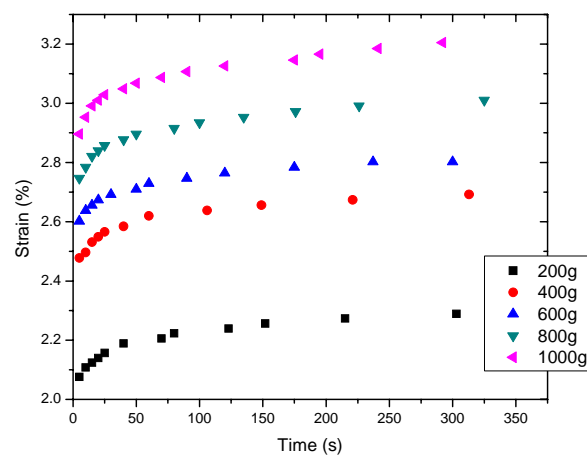


Fig. 2 Experimental data of creep of leather

3.2 Generalized Kelvin model and modified generalized Kelvin model

As discussed above, a single Kelvin unit model may not satisfactorily describe the creep

behaviors of such complex biomaterials as leather. Therefore, generalized Kelvin model with one to four Kelvin units and modified generalized Kelvin model with one to three Kelvin units were applied to fit the creep behavior of samples. Generally chooses the logarithmic coordinate mapping to display the fitting effect and the wide range time dependence.

Fig.3~ Fig.4 show the curves of the sample applied 1000g fitted by generalized Kelvin model with one to four Kelvin units and modified generalized Kelvin model with one to three Kelvin units in a logarithm-coordinate, respectively.

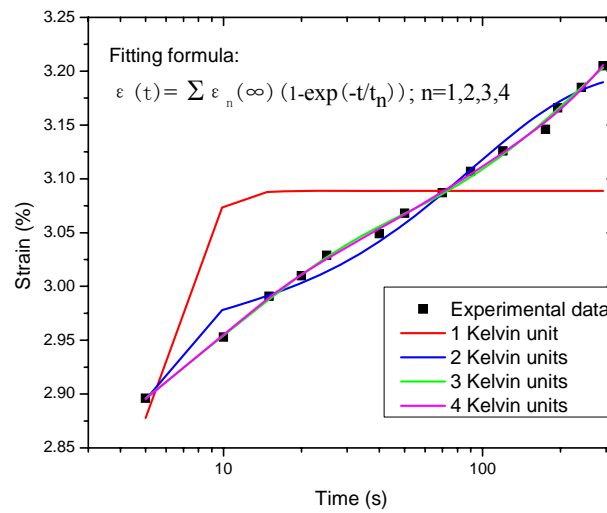


Fig. 3 Curves fitted by generalized Kelvin model with one to four Kelvin units

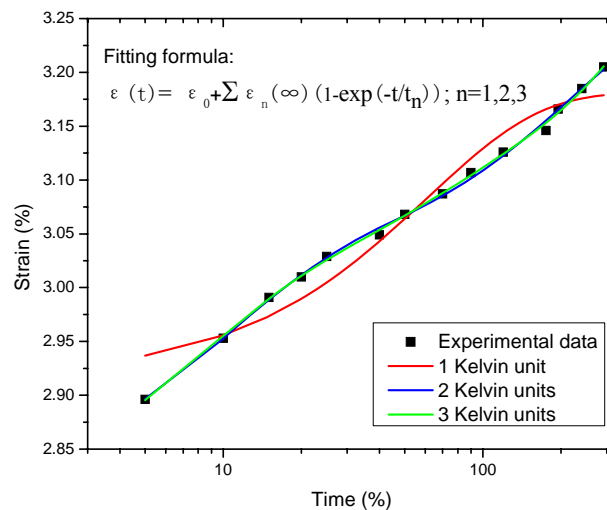


Fig.4 Curves fitted by modified generalized Kelvin model with one to three Kelvin units

In both Figures, the squares represent the experimental data and solid lines are fitted curves. From Fig.3~ Fig.4, it can be found that the accuracy of the fitted curves by the two

models increased markedly with the increase of the number of units. For generalized Kelvin model, the curve of the model with 4 Kelvin units nearly overlaps that of the model with 3 Kelvin units. Regarding to modified generalized Kelvin model, the curve of the model with 3 Kelvin units nearly overlaps that of the model with 2 Kelvin units. It means that the generalized Kelvin model with 3 Kelvin units may describe and imitate the creep behaviors of leather successfully, while the modified generalized Kelvin model with 2 Kelvin units is ok to describe and imitate the creep behaviors of leather successfully.

The determination coefficients of the curves fitted by generalized Kelvin model and modified generalized Kelvin model are shown in Table 1 and Table 2, respectively. Table 1 and Table 2 indicated that the determination coefficient reaches 0.9983 and 0.9982, when the number of Kelvin units of generalized Kelvin model and modified generalized Kelvin model increased to 3 and 2, respectively. When the number of Kelvin units still increases more, no obvious improvement can be found. It is to say that generalized Kelvin model with 3 Kelvin units and modified generalized Kelvin model with 2 Kelvin units can satisfactorily describe the creep behavior of the leather samples. Leathers are composed of cross networks of collagen protein fibers. There are plenty of holes in the network and between collagen molecules, which may vest leather with good elasticity^[9]. When leathers are stretched, an instantaneous deformation will appear. Therefore, when the generalized Kelvin model and the modified generalized Kelvin model are compared, the latter is better to describe the creep behaviors of leather. So the modified generalized Kelvin model with 2 Kelvin units can be a mechanical model of creep behavior of leather.

Table 3 shows both experimental data and the model calculation data of the modified generalized Kelvin model with 1, 2, and 3 Kelvin units. From Table 3, we know that, the modified generalized Kelvin model with two Kelvin units may describe and imitate the creep behavior of samples successfully. Its correlation coefficient (R) obtained is 0.9991. The data of $\varepsilon(t)$ calculated by the model are very close to the corresponding experimental data. It means that the imitation is rather reliable. The more the number of Kelvin units, the better the calculated data coincides to the experimental data, and the less error is. It means that the modified generalized Kelvin model with more Kelvin units can describe the creep behavior of leather more complete. In general, leather contains a number of creep multi-units. Although the limited creep units of materials can describe the creep behavior successfully, actually these are the averages of a series of creep units.

Table 1. Determination coefficients (R^2) of curve fitted with generalized Kelvin models with different numbers of Kelvin units

Number of units	1	2	3	4
R^2	0.3468	0.9838	0.9983	0.9989

Table 2. Determination coefficients (R^2) of curve fitted with modified generalized Kelvin models with different numbers Kelvin units

Number of units	1	2	3
R^2	0.9554	0.9982	0.9989

Table 3. Strain Data of the sample of experimental and calculation by modified generalized Kelvin model

Time (s)	Measurement	Calculation	Error	Calculation	Error	Calculation	Error
	$\varepsilon(t)$ (%)	$\varepsilon(t)$ n=1 (%)	n=1 (%)	$\varepsilon(t)$ n=2 (%)	n=2 (%)	$\varepsilon(t)$ n=3 (%)	n=3 (%)
5	2.896	2.937	-0.041	2.897	-0.001	2.895	0.001
10	2.953	2.956	-0.003	2.953	0	2.955	-0.002
15	2.991	2.974	0.017	2.989	0.002	2.989	0.002
20	3.010	2.990	0.02	3.012	-0.002	3.011	-0.001
25	3.029	3.005	0.024	3.028	0.001	3.026	0.003
40	3.049	3.043	0.006	3.055	-0.006	3.054	-0.005
50	3.068	3.064	0.004	3.067	0.001	3.067	0.001
70	3.087	3.097	-0.01	3.085	0.002	3.088	-0.001
90	3.107	3.120	-0.013	3.101	0.006	3.104	0.003
120	3.126	3.144	-0.018	3.123	0.003	3.124	0.002
175	3.146	3.166	-0.02	3.156	-0.01	3.153	-0.007
195	3.166	3.170	-0.004	3.165	0.001	3.163	0.003
241	3.185	3.176	0.009	3.185	0	3.183	0.002
Average absolute error (%)		0.01454		0.00269		0.00254	
Average relative error		0.479%		0.0798%		0.0776%	

3.3 Rheological constants of the model

Table 4 shows the R^2 of the fitting, equilibrium strains and retardation times of every Kelvin unit. When $t=0$, $\varepsilon(0)=\varepsilon_0$, by which Fig.5 was obtained. It means that leather goods will get a strain of ε_0 at the moment of being dressed. This behavior of leather may guarantee a comfortable feeling for the people who are dressed. When $t=\infty$, $\varepsilon(\infty)=\varepsilon_0+\varepsilon_1(\infty)+\varepsilon_2(\infty)$. It means the strain of leather samples will reach a definite value and will not increase anymore.

So the leather goods will have a good shape-stability. ε_0 increases markedly with the increase of the force applied, and $\varepsilon_1(\infty)$ shows a gradual growing trend. τ_1 changes in a certain range, so τ_1 should be fixed value under different forces applied.

Table 4. Rheological constants and determination coefficients (R^2) of fittings with modified generalized Kelvin model with 2 series Kelvin units

Force (g)	ε_0 (%)	$\varepsilon_1(\infty)$ (%)	τ_1 (s)	$\varepsilon_2(\infty)$ (%)	τ_2 (s)	Determination coefficient (R^2)
200	2.040	0.121	16.245	0.152	161.610	0.9976
400	2.438	0.155	19.919	0.139	250.347	0.9958
600	2.540	0.109	7.309	0.163	95.624	0.9983
800	2.678	0.164	10.688	0.194	158.968	0.9988
1000	2.806	0.211	9.751	0.255	225.939	0.9982

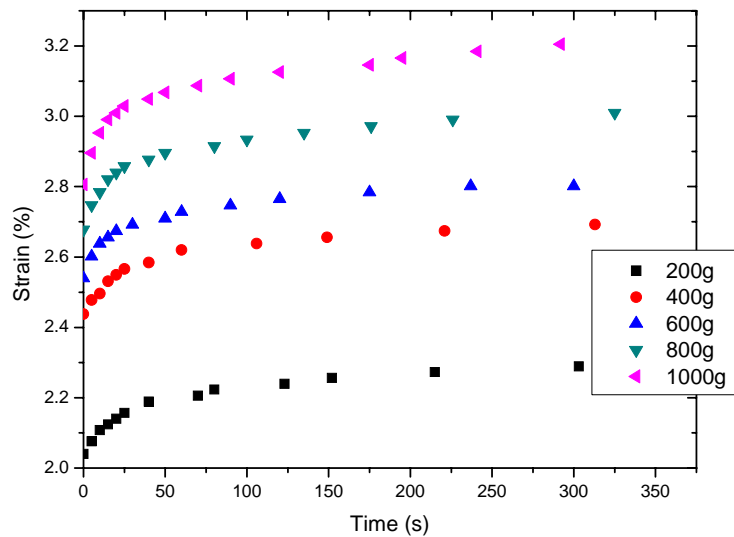


Fig.5 Creep curve of experimental and added data

4 Conclusions

A Lastometer type experiment is employed to obtain the creep data of leather and the creep behavior of the samples were successfully described and simulated by the modified generalized Kelvin models with 2 Kelvin units, which can be the mechanical model of the creep behavior of samples. The comfortable characteristics and shape-stability of leather are described by the rheological constants of the model.

Acknowledgements

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